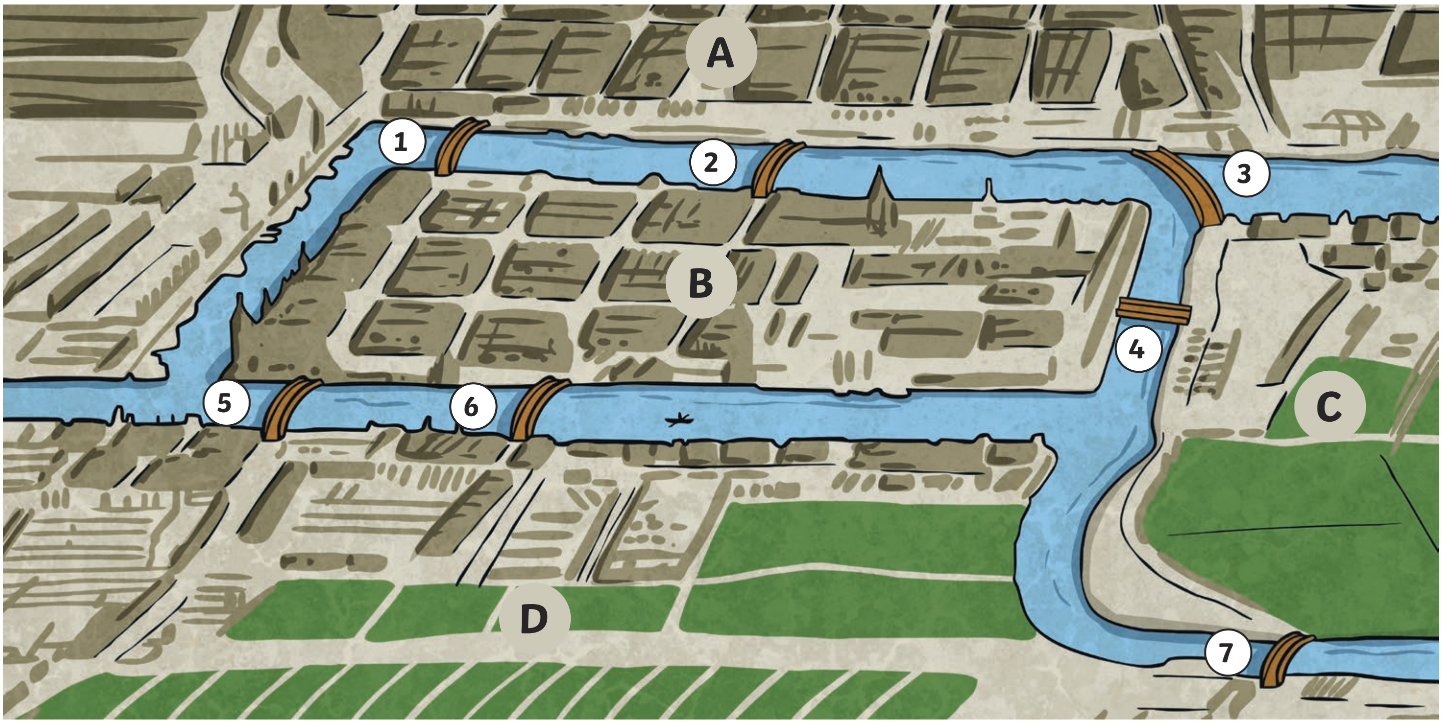
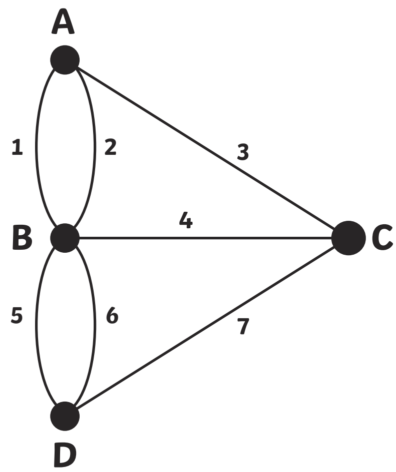
Networks

The city of Königsberg has seven bridges in the old town area. People wondered if they could find a route to cross all 7 bridges without having to cross any bridge more than once.

Can you find a route across all the bridges, only going over each bridge once? In this diagram, the letters indicate the land or islands, the numbers indicate the bridges.



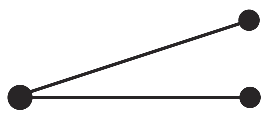
A mathematician called Euler worked on the problem and drew a network to show the paths needed to cross all the bridges.



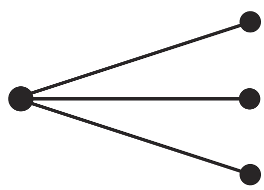
Is it easier to try and traverse the network with this diagram? (Traverse means travel around all the network, only travelling on each arc once.) If you were to traverse the diagram with a pencil, you would not take your pencil from the paper and never go over an arc you have drawn previously.

In a network, the lines are called arcs and the points where the arc meet are called nodes (node) or vertices (vertex).

Euler noticed that each vertex had either an even or odd number of arcs.

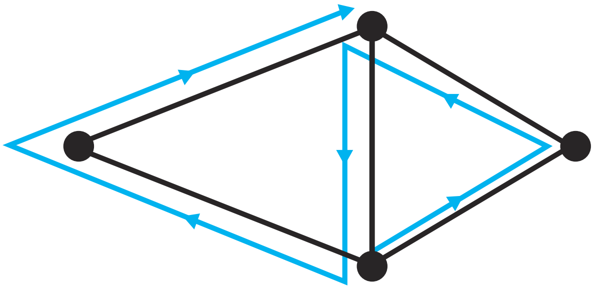


An even vertex has an even number of arcs at the vertex.



An odd vertex has an odd number of arcs at the vertex.

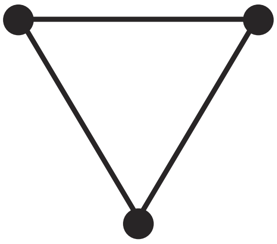
For each of the following networks, find the number of odd and even vertices, and whether the network can be traversed. Remember, to traverse the network, you must keep your pencil on the network and pass over every arc only once. You'll need to find the 'Euler path' around the network to check whether it's traversable. Here's an example:



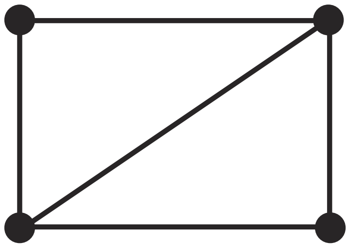
Record your results the table.

Network	Even Vertices	Odd Vertices	Traversable
A			
B			
C			
D			
E			
F			
G			
H			
I			
J			

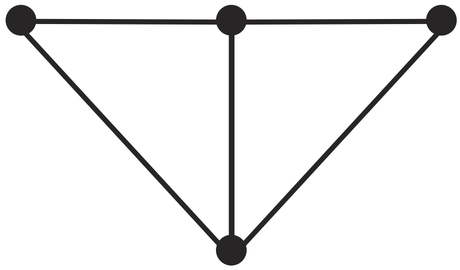
A)



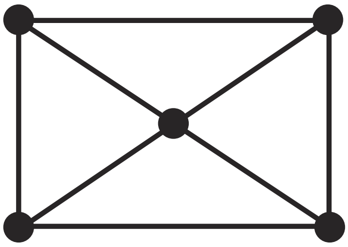
B)



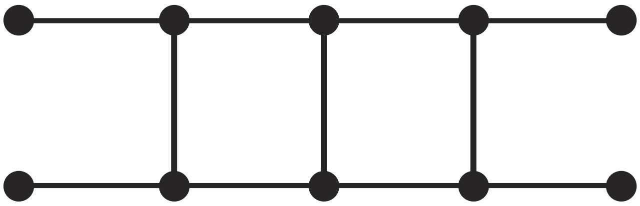
C)



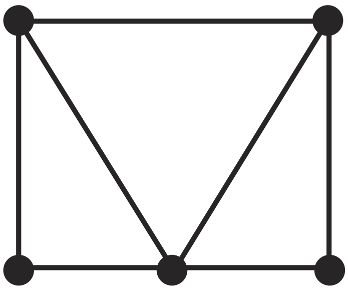
D)



E)

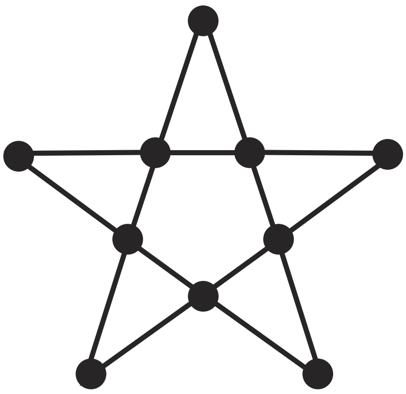


F)

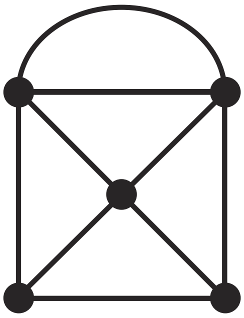


G)

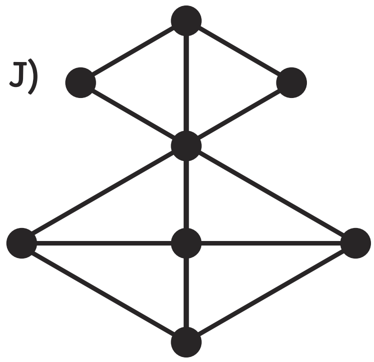
H)



I)



J)



How can you use the number of odd and even vertices to tell if a network is traversable? Can you make a rule for whether a network is traversable or not?

Apply your idea to the seven bridges of Konigsburg.

Create some networks of your own for a partner to test. Make some traversable and some not, using the rule you have discovered.

Networks **Answers**

Network	Even Vertices	Odd Vertices	Traversable
A	3	0	Yes
B	2	2	Yes
C	2	2	Yes
D	1	4	No
E	0	10	No
F	3	2	Yes
G	2	4	No
H	10	0	Yes
I	3	2	Yes
J	4	4	No

A network can be traversed if it has none or 2 odd vertices.

The network for the bridges of Konigsburg has 4 odd vertices so is not traversable.