Maths Calculation Policy
Concrete / Pictorial / Abstract

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy.

Ratified by the Headteacher July 2020

Recommended practice delivering a mastery approach

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Evidence repeatedly shows that mixed ability seating increases less confident pupils' perception of mathematical capability, which impacts positively upon outcomes. While not a school policy, it is recommended to avoid ability groups. This presents a challenge in ensuring the more confident mathematicians are being extended. An extension tasks to deepen understanding is the most simplistic way around this. Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

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Addition and Subtraction Vocabulary

Year 3

addition
add, more, and
make, sum, total
altogether
double
near double
half, halve
one more, two more … ten more … one hundred more
how many more to make …?
how many more is … than …?
how much more is …?
subtract take away how many are left/left over?
how many have gone?
one less, two less, ten less … one hundred less
how many fewer is … than …?
how much less is …?
difference between
equals
is the same as
number bonds/pairs/facts
missing number
tens boundary, hundreds boundary

Year 4

addition
add, more, and
make, sum, total
altogether
double
near double
half, halve
one more, two more … ten more … one hundred more
how many more to make …?
how many more is … than …?
how much more is …?
subtract take away how many are left/left over?
how many have gone?
one less, two less, ten less … one hundred less
how many fewer is … than …?
how much less is …?
difference between
equals
is the same as
number bonds/pairs/facts
missing number
tens boundary, hundreds boundary
inverse

Addition and Subtraction Vocabulary

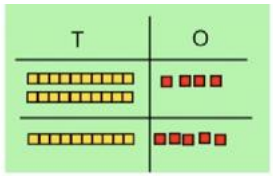
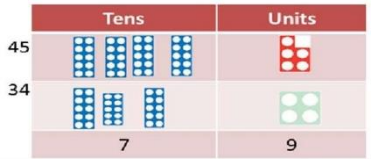
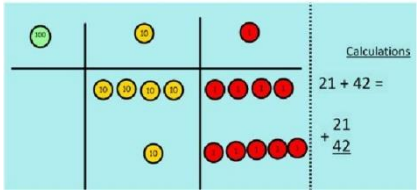
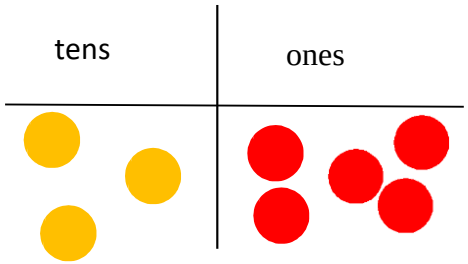
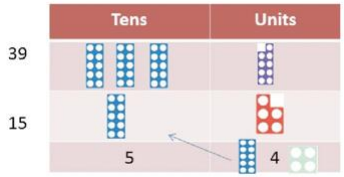
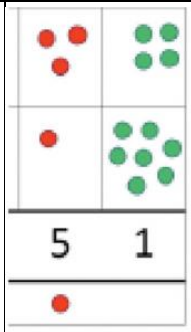
Year 5

addition
add, more, and
make, sum, total
altogether
double
near double
half, halve
one more, two more … ten more … one hundred more
how many more to make …?
how many more is … than …?
how much more is …?
subtract take away how many are left/left over?
how many have gone?
one less, two less, ten less … one hundred less
how many fewer is … than …?
how much less is …?
difference between
equals
is the same as
number bonds/pairs/facts
missing number
tens boundary, hundreds boundary, ones boundary, tenths boundary

Year 6

addition
add, more, and
make, sum, total
altogether
double
near double
half, halve
one more, two more … ten more … one hundred more
how many more to make …?
how many more is … than …?
how much more is …?
subtract take away how many are left/left over?
how many have gone?
one less, two less, ten less … one hundred less
how many fewer is … than …?
how much less is …?
difference between
equals
is the same as
number bonds/pairs/facts
missing number
tens boundary, hundreds boundary, ones boundary, tenths boundary

ADDITION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 3 **NC: Add numbers with up to 3 digits, using formal written methods of columnar addition** **Column addition – start off with no regrouping**	 Dienes or numicon Add together the ones first, then the tens.   Moving to using place value counters	Children move to drawing the counters using a tens and one frame. 	2 2 3 + 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds.
Move onto column addition with regrouping	 Exchange ten ones for a ten. Model using numicon and place value counters.	 3 4 +1 7 **C**hildren can draw a representation of the grid to further support their understanding, carrying the ten	20 + 5 40 + 8 60 + 13 = 73

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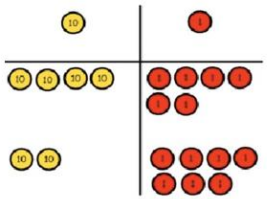
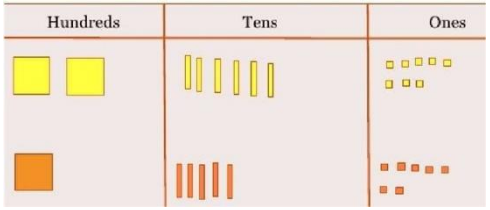
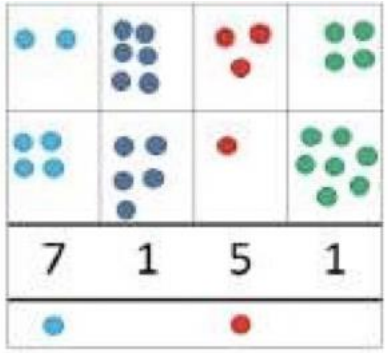
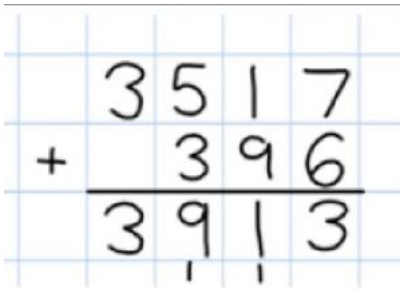
	 $46+27=73$	<u>underneath</u> the line.	Start by partitioning the numbers before formal column to show the exchange. $\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline 1\ 1 \end{array}$
Estimate the answers to questions and use inverse operations to check answers.	 Estimating 98 + 17 = ? 100 + 20 = 120	Use number lines to illustrate estimation. 	Building up known facts and using them to illustrate the inverse and to check answers. 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

ADDITION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 4 **NC: Estimate and use inverse operations to check answers to a calculation.**	**As per Y3** Estimating 98 + 17 = ? 100 + 20 = 120 	**As per Y3** Use number lines to illustrate estimation. 	**As per Y3** Building up known facts and using them to illustrate the inverse and to check answers. 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18
NC: Add numbers with up to 4 digits using the formal written methods of columnar addition	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using place value grid	 Continue from previous work to carry hundreds as well as ten. Relate to money and measures.
YEAR 5 **NC: Estimate and use inverse operations to check answers to a calculation.**	**As per Y3** Estimating 98 + 17 = ? 100 + 20 = 120 	**As per Y3** Use number lines to illustrate estimation. 	**As per Y3** Building up known facts and using them to illustrate the inverse and to check answers. 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

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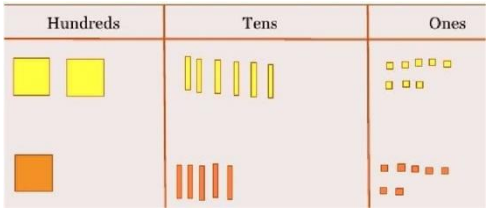
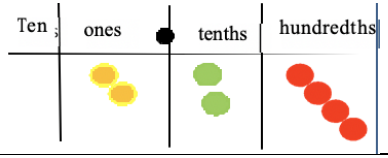
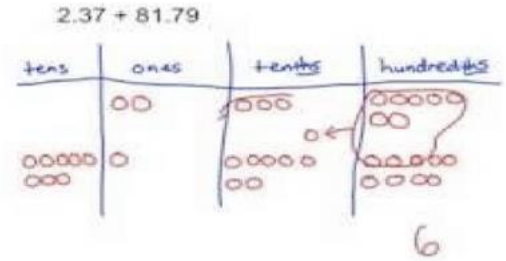
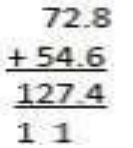
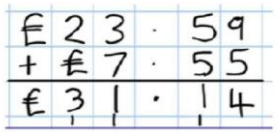
NC: Add whole numbers with more than 4 digits, including using formal written methods **Add decimals with 2 decimal places, including money.**	**As year 4** Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.  Introduce decimal place value counters and model exchange for addition.  	 	 
YEAR 6 **NC: Estimate and use inverse operations to check answers to a calculation.**	**As per Y3** Estimating 98 + 17 = ? 100 + 20 = 120 	**As per Y3** Use number lines to illustrate estimation.  	**As per Y3** Building up known facts and using them to illustrate the inverse and to check answers. 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

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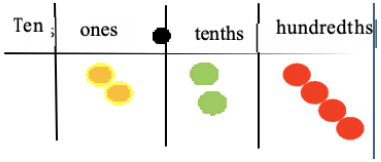
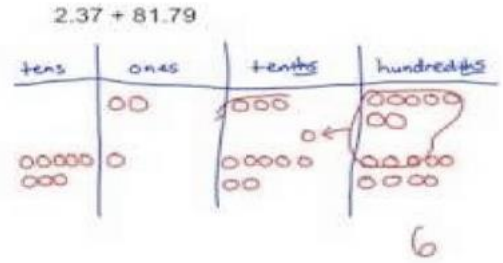
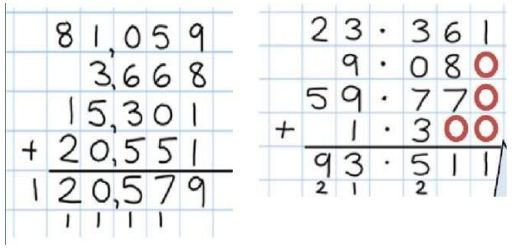
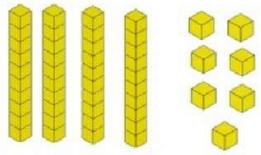
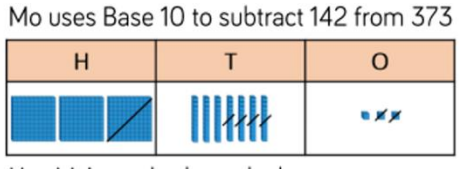
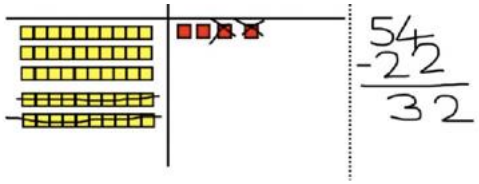
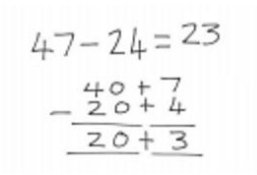
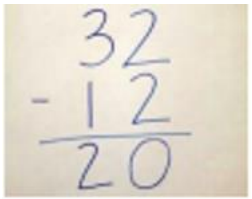
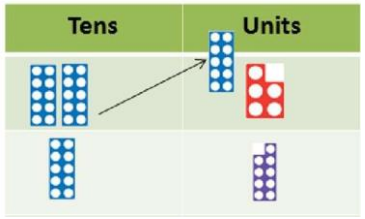
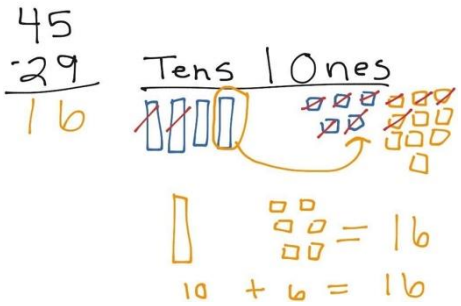
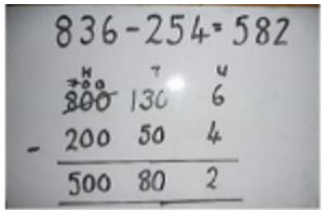
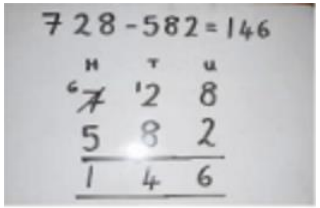
NC: Add several numbers of increasing complexity, including money, measure and decimals with different numbers of decimal places.	**As per Y5** Introduce decimal place value counters and model exchange for addition. 	**As per Y5** 	Insert zeros for place holders. 

SUBTRACTION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 3 **NC: Subtract numbers with up to 3 digits, using formal written methods of columnar subtraction** **Start with column subtraction without regrouping**	 47 – 32 Use base 10 or Numicon to model 	 Draw representations to support understanding	 Intermediate step may be needed to lead into clear subtraction understanding. 
Move onto column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones.	 Children may draw base 10 or pv counters and cross off	 Begin by partitioning into place value columns  Then move onto formal method

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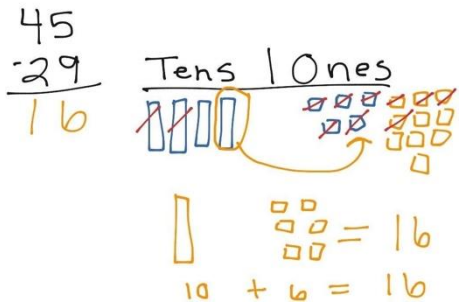
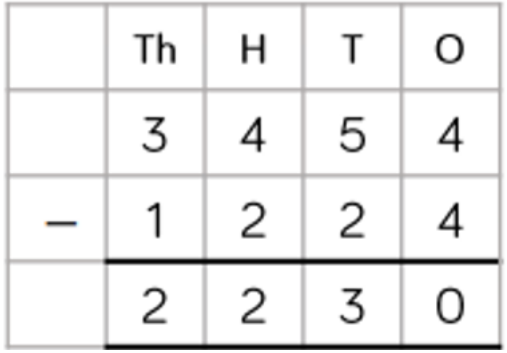
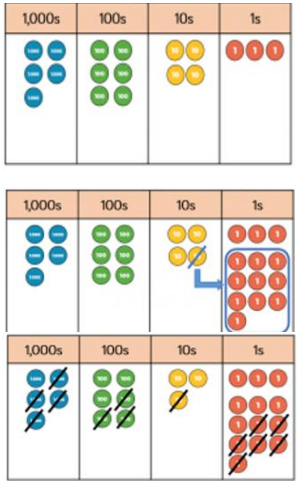
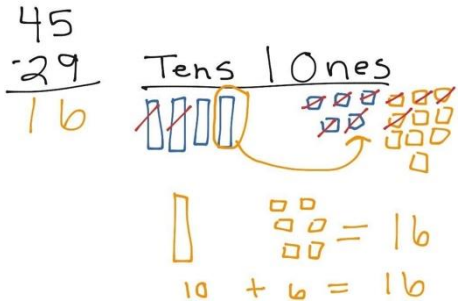
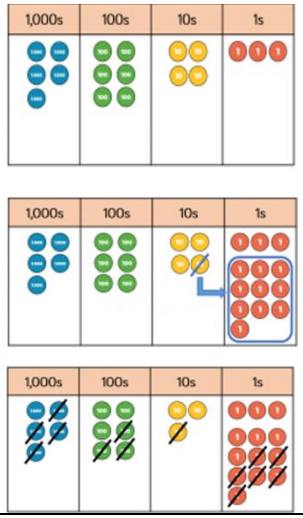
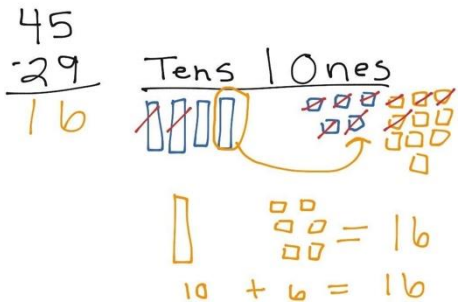
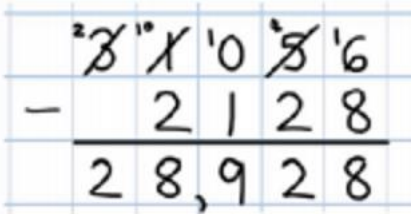
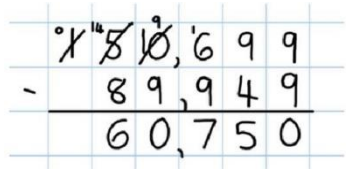
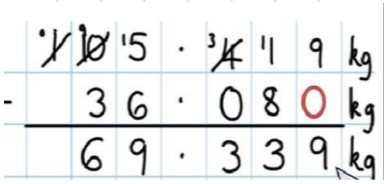
SUBTRACTION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 4 **NC: Subtract numbers with up to 4 digits using the formal written methods of subtraction where appropriate** **Start with no exchange.**	3453 – 1224 = 	Children to draw pv counters cross off. **See Y3**. 	
As above but with one exchange. Then move onto more than one exchange.	 5643-4316= Model process of exchange using Numicon, base ten and then move to pv counters.	Children to draw pv counters and show their exchange. **See Y3**. 	 Use the phrase ‘take and make’ for exchange.

SUBTRACTION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 5 **NC: Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)** **Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal up to 3 decimal places**	**As year 4.** 5643-4316= Model process of exchange using Numicon, base ten and then move to pv counters 	Children to draw pv counters and show their exchange. **See Y3**. Especially when problem solving. 	 Use zeros for placeholders. 
YEAR 6 **NC: Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal places)**	**As year 4** 5643-4316= Model process of exchange using Numicon, base ten and then move to pv counters	Children to draw pv counters and show their exchange. See Y3. Especially when problem solving.	 

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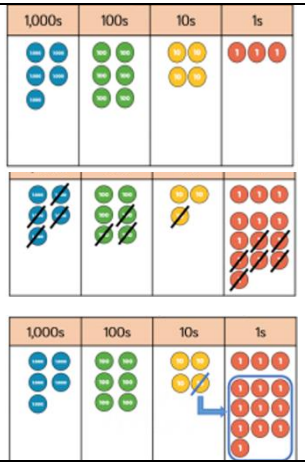
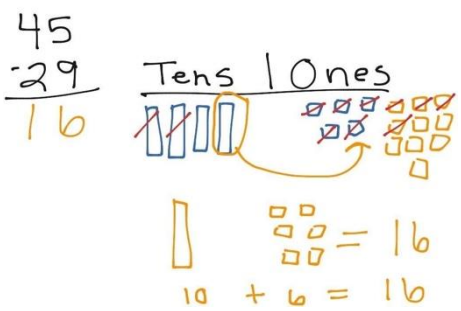
	 1,000s 100s 10s 1s 1,000s 100s 10s 1s	 45 -29 16 Tens | Ones = 16 10 + 6 = 16	

Multiplication and Division Vocabulary

Year 3

multiplication
multiply
multiplied by
multiple, factor
groups of
times
product
once, twice, three times ... ten times
repeated addition
division
dividing, divide, divided by, divided into
left, left over, remainder
grouping
sharing, share, share equally
one each, two each, three each ... ten each
group in pairs, threes ... tens
equal groups of
doubling
halving
array
row, column
number patterns
multiplication table
multiplication fact, division fact

Year 4

multiplication
multiply
multiplied by
multiple, factor
groups of
times
product
once, twice, three times ... ten times
repeated addition
division
dividing, divide, divided by, divided into
left, left over, remainder
grouping
sharing, share, share equally
one each, two each, three each ... ten each
group in pairs, threes ... tens
equal groups of
doubling
halving
array
row, column
number patterns
multiplication table
multiplication fact, division fact
inverse
square, squared
cube, cubed

Multiplication and Division Vocabulary

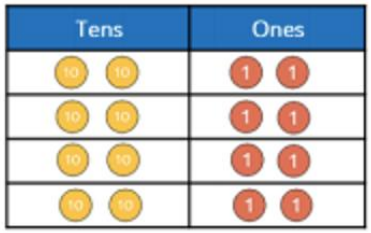
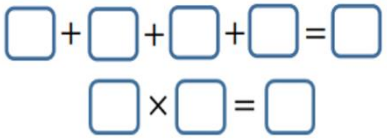
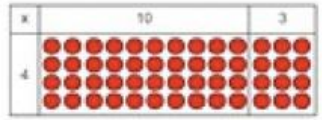
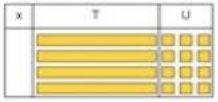
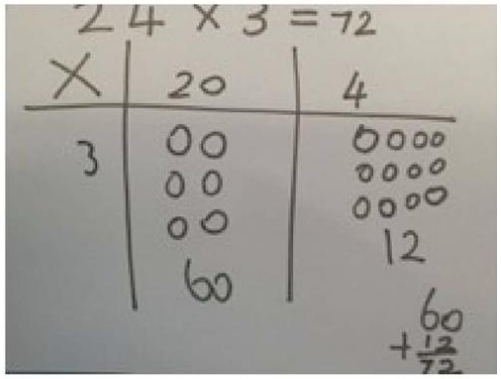
Year 5

multiplication
multiply
multiplied by
multiple, factor
groups of
times
product
once, twice, three times ... ten times
repeated addition
division
dividing, divide, divided by, divided into
left, left over, remainder
grouping
sharing, share, share equally
one each, two each, three each ... ten each
group in pairs, threes ... tens
equal groups of
doubling
halving
array
row, column
number patterns
multiplication table
multiplication fact, division fact
inverse
square, squared
cube, cubed

Year 6

Multiplication
multiply
multiplied by
multiple, factor
groups of
times
product
once, twice, three times ... ten times
repeated addition
division
dividing, divide, divided by, divided into
left, left over, remainder
grouping
sharing, share, share equally
one each, two each, three each ... ten each
group in pairs, threes ... tens
equal groups of
doubling
halving
array
row, column
number patterns
multiplication table
multiplication fact, division fact
inverse
square, squared
cube, cubed

MULTIPLICATION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 3 **NC: Multiply 2 digit numbers by 1 digit numbers** **Start with repeated addition, then the grid method, progressing to the formal method**	Repeated addition. 22 x 4 =  Show link to multiplication.  Show the links with arrays to first introduce. the grid method  4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method.  4 rows of 13	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.  | ×	30	5
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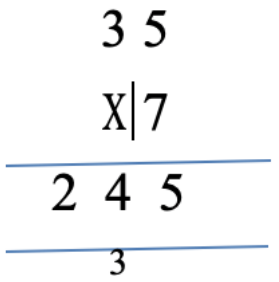
7	210	35	 **210 + 35 = 245** Move forward to the formal written method.  35 X 7 245 3

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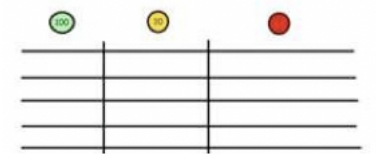
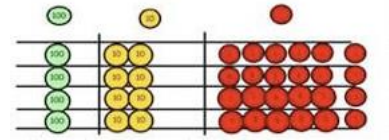
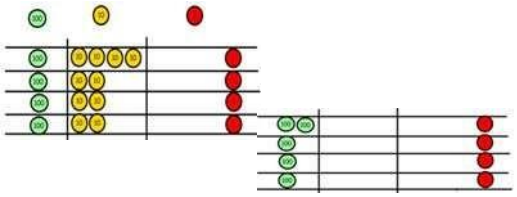
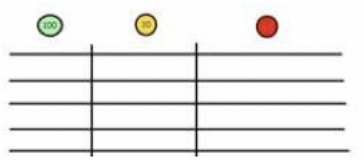
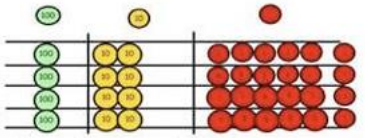
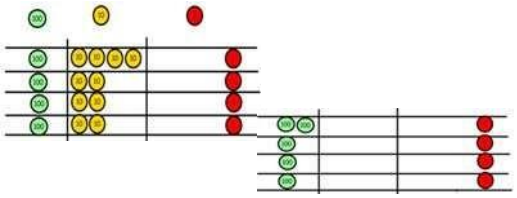
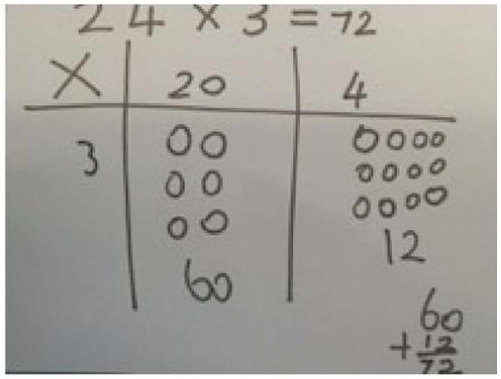
	Move on to place value counters to show how we are finding groups of numbers. We are multiplying by 4 so we need 4 rows.  4 x 126=  4x126= Fill each row with 126. Add up each column, starting with the ones and making any exchanges needed. Then you have your answer. 		

MULTIPLICATION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 4 **Grid method recap from year 3 for 2 digits multiplied by 1 digit.**	Use place value counters to show how we are finding groups of numbers. We are multiplying by 4 so we need 4 rows.  4 x 126=  4x126= Fill each row with 126. Add up each column, starting with the ones and making any exchanges needed. Then you have your answer. 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | 7 | 210 | 35 | 210 + 35 = 245 Move forward to the formal written method. 35 X 7 245 3

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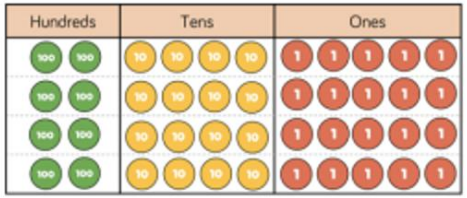
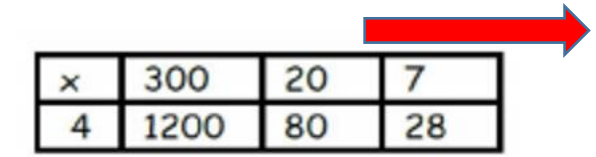
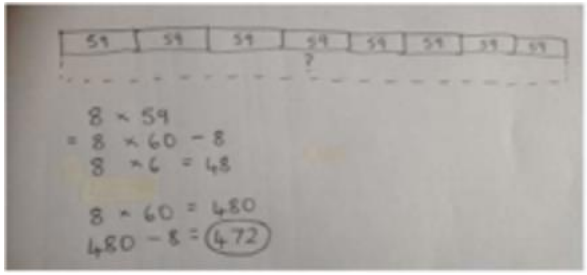
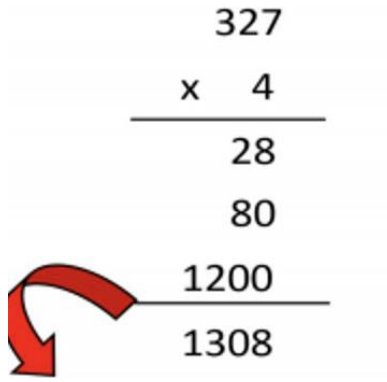
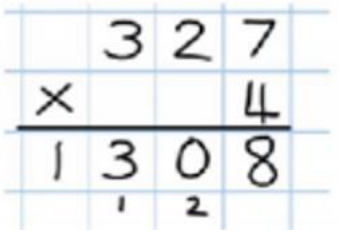
NC: Multiply 2 and 3 digit numbers by a one digit number using formal written layout.	Build on previous steps to represent a three-digit number multiplied by a one-digit number with concrete manipulatives 245 x 4 = 	Children can represent their work with place value counters in a way that they understand. They use their knowledge of exchanging ten ones for one ten in addition and apply this to multiplication, including exchanging multiple groups of 10. 	 Move from expanded to compact method. 

MULTIPLICATION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 5 **NC: Multiply numbers up to 4 digits by a one or two-digit number using a formal written method,** including long multiplication for 2 digit numbers.	Children build on previous steps to represent a 4-digit number multiplied by a 1-digit number using concrete manipulatives. Then move onto multiplication with exchange in one and then more than one column. 3023 x 3 =  It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at this stage of multiplication.	The grid method may be used to show how this relates to a formal written method.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	 This may lead to the compact method. 

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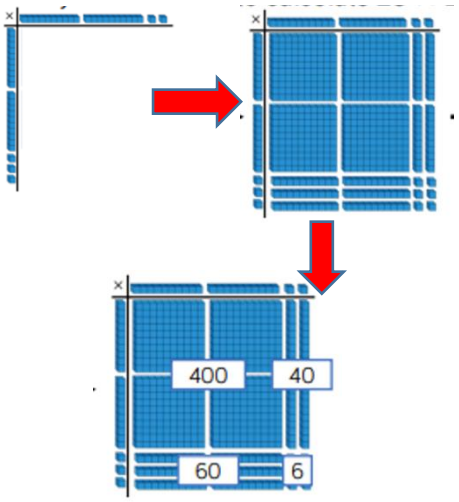
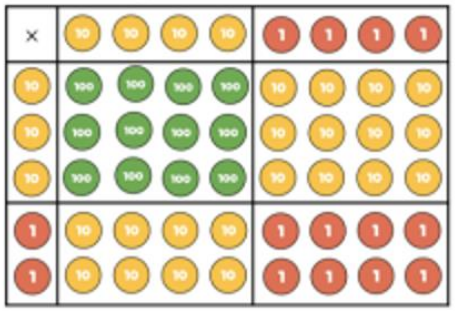
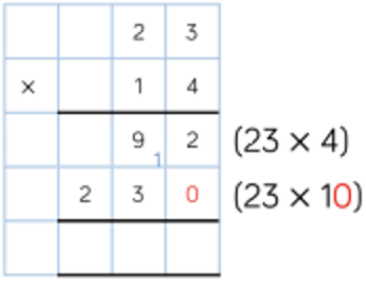
NC: Multiply numbers up to 4 digits by a one or two-digit number using a formal written method, **including long multiplication for 2 digit numbers.**	Children use Base 10 to represent the area model of multiplication which will enable them to see the size and scale linked to multiplication. 23 x 22 =  	They then move onto representing multiplication more abstractly with place value counters. These can also be drawn. 44 x 32 = 	And then with numbers. 44 x 32 = × | 40 | 4 30 | 1,200 | 120 2 | 80 | 8 Children will move on from the area model and work towards more formal methods. They will start by exploring the role of the zero in the column method and understanding its importance.  Children will extend their multiplication skills to multiplying 3-digit numbers by 2-digit numbers. Methods such as the 'area' model are still useful.

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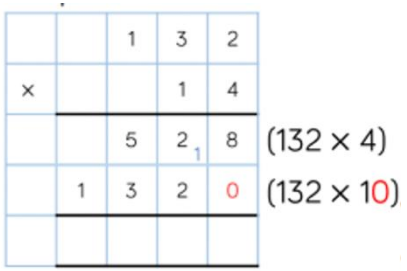
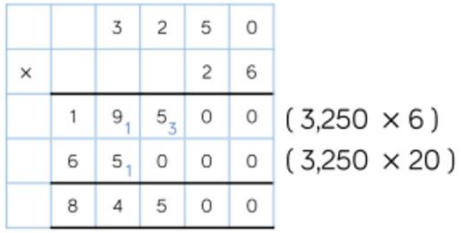
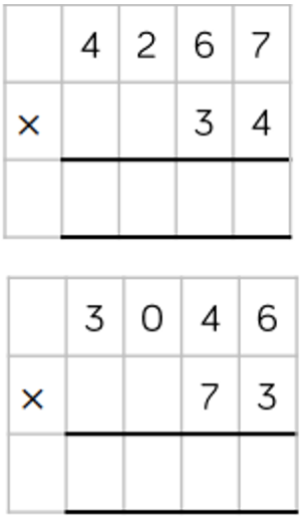
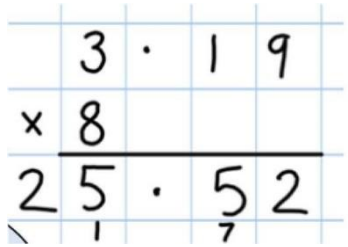
			 Children will extend their multiplication skills to multiplying 4-digit numbers by 2-digit numbers. Methods such as the 'grid' model are still useful. It is important that children understand the steps taken when using this multiplication method. 

MULTIPLICATION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 6 **NC: Multiply multi-digit numbers up to 4 digits by 2 digits using formal written method of long multiplication.**	Children build on their knowledge of column multiplication. It may be useful to revise multiplication by a single digit first, then 2- and 3-digit numbers before moving on when ready to the largest calculations. Manipulatives may still be used with the corresponding long multiplication modelled alongside. See previous year groups.	 Continue to use bar models to support problem solving. See previous year groups.	 See previous year groups.
NC: Multiply one-digit numbers with up to two decimal places by whole numbers.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

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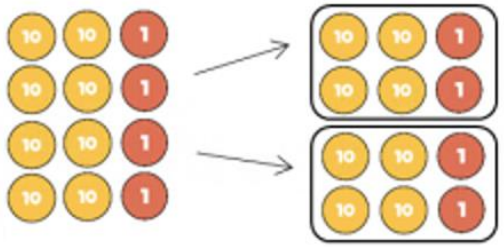
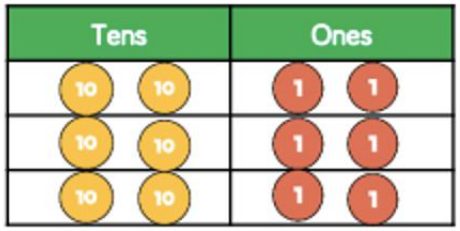
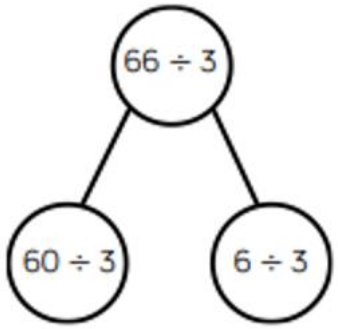
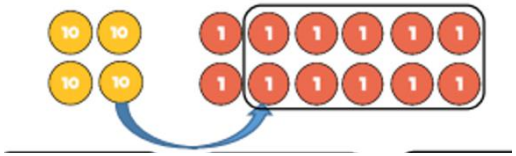
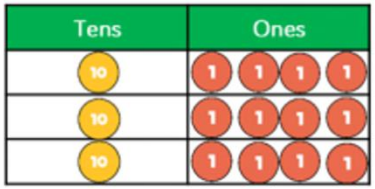
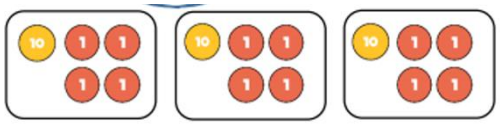
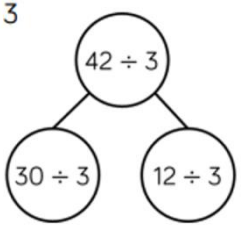
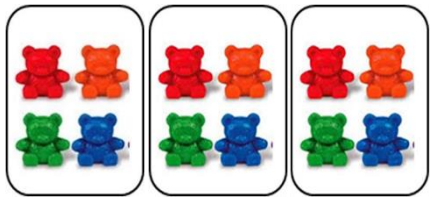
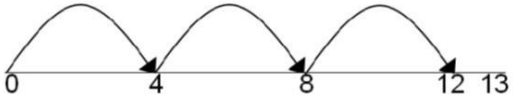
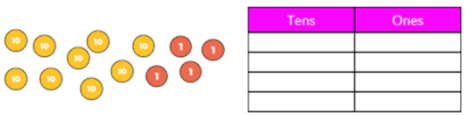
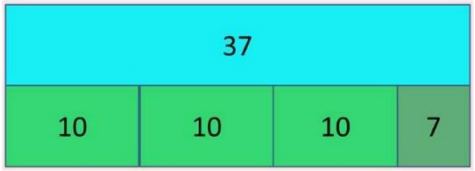
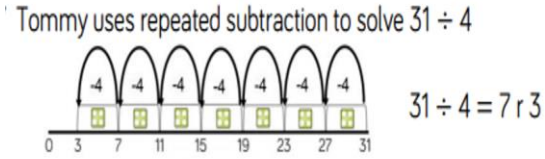
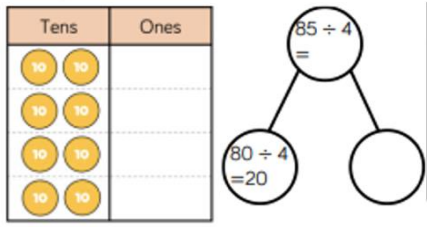
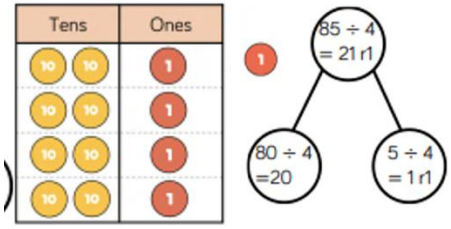
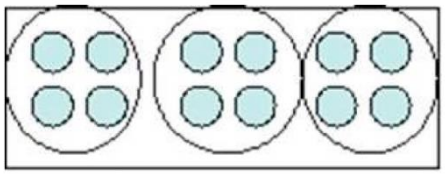
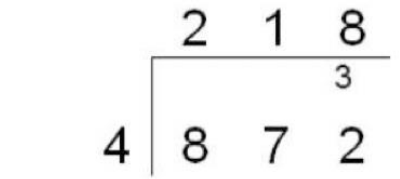
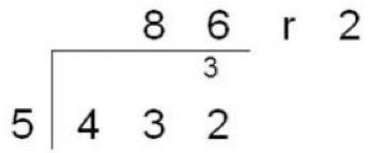
DIVISION

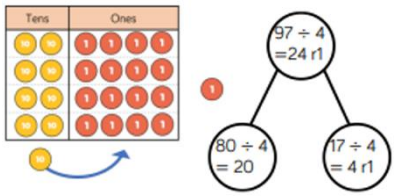
Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 3 **NC: Divide a 2-digit number by a 1 digit number.** **Start with dividing numbers that do not require an exchange or remainders.**	Use place value counters to divide numbers. It is important that children divide the tens first and then the ones. 84 ÷ 2 = 	Link the number of rows in their place value chart with the number they are dividing by.   	69 ÷ 3 = 96 ÷ 3 = 86 ÷ 2 =
NC: Divide a 2-digit number by a 1 digit number. **Move on to dividing numbers that require an exchange between the tens and ones but no remainders.**	42 ÷ 3 =  Use place value counters to divide 42 into 3 equal groups. Share the tens first	42 ÷ 3 = Use a similar method with a place value chart. Sharing the tens first and then exchanging the ten for ones.  	96 ÷ 8 = 96 ÷ 4 = 96 ÷ 3 = 96 ÷ 6 =

	and exchange the remaining ten for ones. Then share the ones. 	42 by 3 	
NC: Divide a 2-digit number by a 1 digit number with remainders.	Make links between division and repeated subtraction which is revision from Y2. 14 ÷ 3 = Divide objects between groups and see how much is left over.  	There are many different pictorial examples that may support children's understanding: 1. Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13÷4 = 3 r 1  2. Draw dots and group them to divide an amount and clearly show a remainder.	Complete written divisions and show the remainder using r. 

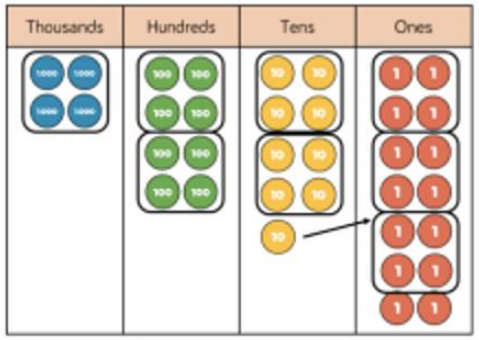
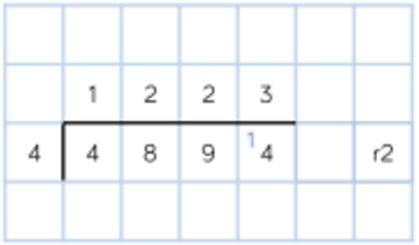
	94÷4 =  Do you need to exchange any tens for ones? Is there a remainder?	 3. Use bar models to show division with remainders.  **This notation is new to Year 3 and will need clear explanation.**  	

DIVISION

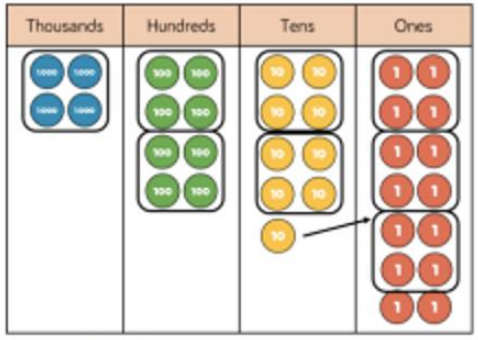
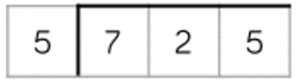
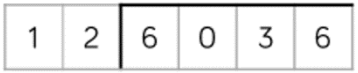
Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 4 **NC: Recall multiplication and division facts for multiplication tables up to 12 × 12.** **Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers** **NO REQUIREMENT IN Y4 NATIONAL CURRICULUM TO DIVIDE USING FORMAL METHODS** Continue to use place value counters to divide in order to explore	Use place value counters to divide using the bus stop method alongside. 85÷4 = Divide the tens first. We are sharing 80 into 4 groups. We can put two tens in each group.  Then divide the ones. We are sharing 5 ones into 4 groups. There is one left over. 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Non-statutory guidelines say that pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers.  

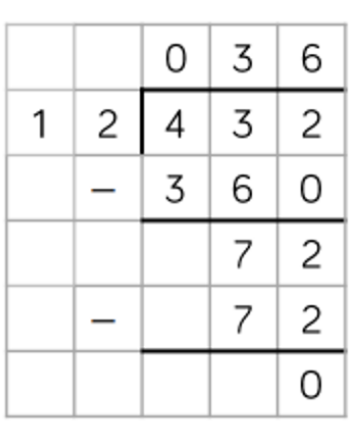
where there are remainders.	Move onto questions which require an exchange. 97÷4 = Start with the biggest place value. We are sharing 90 into 4 groups. We can out 2 tens in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups. We look how much is in 1 group so the answer is 24 remainder 1.		

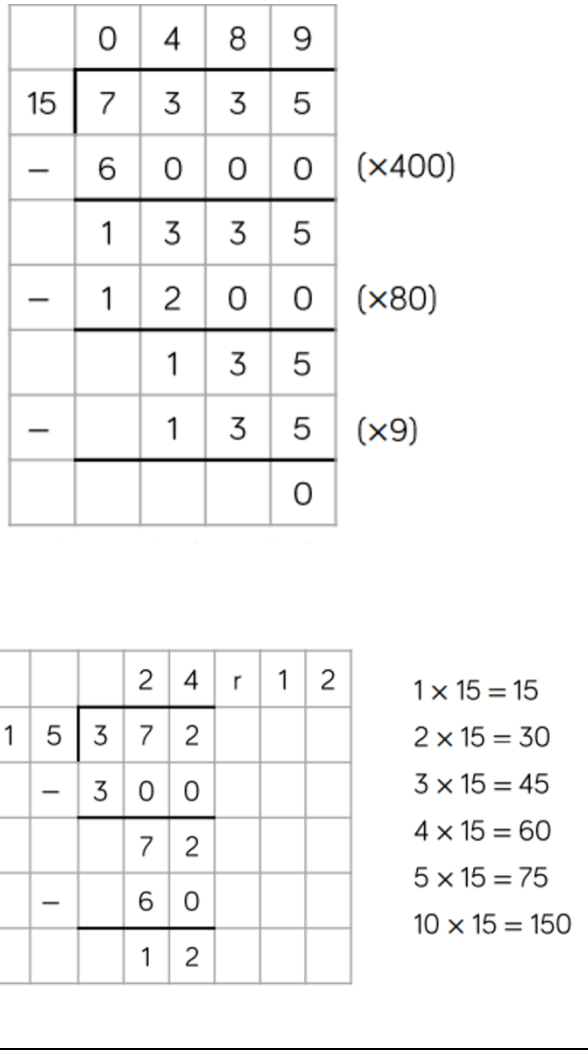
DIVISION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 5 **NC: Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**	Continue to use place value counters to partition and then group their number to further develop their understanding of short division. Start to focus on understanding remainders in context. 4,894 ÷ 4 =  With this method, the children group the counters in the columns according to the divisor and exchange where necessary.	Children continue to use drawn diagrams with dots or circles to help them divide numbers.	Model the bus stop method alongside the concrete and pictorial so children can see the link. 

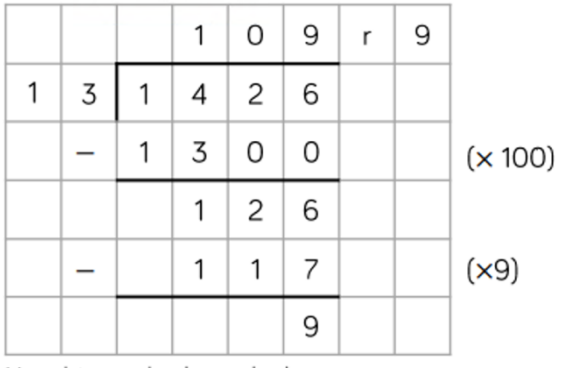
DIVISION

Objective/Strategy	Concrete	Pictorial	Abstract
YEAR 6 **Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context**	Children build on their understanding of dividing up to 4 digits by 1 digit, by now dividing by up to two digits. Focus on the grouping structure of division. **As per Year 5** Continue to use place value counters to partition and then group their number to further develop their understanding of short division. Start to focus on understanding remainders in context. 4,894 ÷ 4 =  With this method, the children group the counters in the columns according to the divisor and exchange where necessary.	Teachers may encourage children to list multiples of the divisor to help them solve the division more easily.	 

NC: Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	Children are introduced to long division as an alternative method of dividing by a two-digit number. They divide three and four digits by two digits with and without remainders, starting with an expended method (with multiples shown) before progressing to the more formal long division method.		 List mutiples of the divisor. 12 × 1 = 12 12 × 2 = 24 12 × 3 = 36 12 × 4 = 48 12 × 5 = 60 12 × 6 = 72 12 × 7 = 84 12 × 8 = 96 12 × 7 = 108 12 × 10 = 120  Long division no remainder

0 4 8 9
15 7 3 3 5
– 6 0 0 0 (×400)
1 3 3 5
– 1 2 0 0 (×80)
1 3 5
– 1 3 5 (×9)
0
2 4 r 1 2
1 5 3 7 2
– 3 0 0
7 2
– 6 0
1 2
1 × 15 = 15
2 × 15 = 30
3 × 15 = 45
4 × 15 = 60
5 × 15 = 75
10 × 15 = 150

			 1 0 9 r 9 1 3 1 4 2 6 – 1 3 0 0 (× 100) 1 2 6 – 1 1 7 (×9) 9